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Soviet Mathematics and Dialectics in the Post-Stalin Era: New Horizons

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The post-Stalin period in Soviet history (1953–1991) was a time of weakening of officia ideological interference with the professional activity of the Soviet scientifi community. Many branches of mathematics—particularly cybernetics, mathematical logic, and the foundations of mathematics—played an important role in the struggle against Stalinist ideological and philosophical dogmatism. The Soviet "constructivist" orientation in the foundations of mathematics expressed a conciliatory attitude toward intuitionism, logicism, and formalism, the main currents in Western mathematics. Dialectical materialism became more fl xible, but it continued to be the only officiall recognized philosophy in the Soviet Union. © 2002 Elsevier Science (USA)

Послесталинский период в советской истории (1953–1991)—время постепенного ослабления официального идеологического вмешательства в жизнь советского научного сообщества. Многие ветви математики—особенно кибернетика, математическая логика и основания математики—играли важную роль в борьбе против сталинского идеологического и философского догматизма. Советский "конструктивистский" подход к основаниям математики выражался в примирительном отношении к интуиционизму, логицизму и формализму, главным течениям в западной математике. Диалектический материализм стал более гибким, но оставался единственной официально признанной философией в Советском Союзе. © 2002 Elsevier Science (USA)

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After the death of Stalin and the demise of the ethnocentric anticosmopolitan campaign in 1953, interest in the history of mathematics acquired a new meaning and wider scope. The study of national traditions ceased to be the exclusive concern of historical scholarship in mathematics; increasingly more attention was given to a systematic study of the great men and traditions in Western mathematics. C. F. Gauss, G. Cantor, F. Klein, H. Grassmann, E. Galois, and other pioneers of modern mathematics were subjects of careful and sympathetic monographic studies. And so were some mathematicians from earlier generations—such as J. Napier, the 17th-century inventor of logarithms. The new trend became evident after the acceptance of cybernetics in 1956 and the inauguration of a concerted effort to widen the base and sharpen the vision of mathematics. Whereas earlier historians placed primary emphasis on the independence and pristine originality of Russian mathematical thought, now they devoted much more attention to Russian scientifi thought as part of the universal search for objective knowledge and as Russia's response to new developments in Western science.

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But the transition to the new outlook was not smooth. A few defenders of Stalinist orthodoxy had much difficulty in sensing the massive outburst of new thought and continued to explore and ramify the Stalinist hybrid of inflated nationalism and rigidly formulated dialectical materialism. E. Kol'man, one of the most obdurate champions of anticosmopolitanism, wrote as late as 1957 of the decadence of Western idealism in mathematics and of the sound realism of Russian and Soviet mathematical thought. He examined the major Western orientations, as represented by Russell's logicism, Poincaré's conventionalism, and Weyl's intuitionism, and charged them once again with isolation from reality and sterile dedication to a mystical view of the universe [Kol'man 1957, 228–236]. It is true, he said, that the great mathematicians of the era of the French Revolution—J. d'Alembert, P. S. Laplace, and G. Monge, who occupied themselves simultaneously with "mathematics and mechanics, physics and natural science"—were materialists, even though of nondialectical bent. But in the totality of Western thought, they were an exception, rather than the rule. Following them, "only a few mathematicians, typified by C. Hermite and F. Klein, showed some inclination toward materialism, but even they sank in the mire of Kantian, conventionalist and similar ideas" [Kol'man 1957, 197]. In J. von Neumann's game theory he saw the most bizarre manifestation of the dwindling fortunes of mathematical idealism. By contrast, Kol'man detected profound and consistent dedication to true science by all the leading Russian mathematicians since the time of N. I. Lobacheskii and M. V. Ostrogradskii. Kolman's anti-Western tirades were the last remnants of crumbling anticosmopolitanism and a telling expression of the basically conservative (and restrictive) role played by Soviet philosophers in the progress of science. At the same time the community of mathematicians had been deeply committed to a new philosophy that encourged a sustained effort to reintegrate Soviet scientific thought into universal science unmarred by nationalist parochialism.

Marxist philosophers were particularly slow in making their opposition toward neopositivism less uncompromising. Meeting in the summer of 1957, social science department heads in institutions of higher education were treated to a sweeping attack on neopositivism as a philosophy of science based on logic and mathematics. The keynoter observed that Russell's logicism, presented as a variety of neopositivist thought, was derailed by Gödel's proof that no powerful mathematical system can achieve the level of full axiomatization by relying on its inner logic. On a more general level, neopositivists were attacked for their alleged tendency "to universalize mathematics"—to interpret the possibility of applying mathematical methods to the full spectrum of scientific knowledge as proof that there is no difference between nature and human society as realms of scientific inquiry. The neopositivists did not recognize the "primacy" of physical reality over psychological reality [Fataliev 1958, 331].

Reinterpretations of the cultural matrix and epistemological make-up of science, evolving at a steady pace during the 1960s–1980s, did not signify the abandonment of basic premises of the Marxist philosophy of mathematics—such as the empirical and social origin of all mathematical theorems and axioms, the primary role of practice in determining the social value of mathematical contributions, the dialectical dynamics of the growth of mathematical knowledge, and the supreme authority of mathematics in explaining the ontological unity of nature. The new trend led to a broader and more flexible interpretation of these and newly added principles. In particular, these reinterpretations produced a more positive attitude toward the contributions of leading Western orientations in the foundations of mathematics. The combination of the rapid expansion of mathematical studies and the loosening of ideological norms led to a lively search for a comprehensive Marxist philosophy of mathematics that had not existed previously—and to a considerable variation and even playfulness in the interpretation of basic principles.

All this took place at the time of a major upsurge in the growth of new branches of mathematics and of a profound restructuring of accumulated mathematical wisdom. Mathematics, we were now told, had entered a third stage in its growth as a systematic body of knowledge. The first stage, according to Soviet commentators, lasted from ancient times to the 17th century and was characterized by a continuous reliance on elementary operations, whether conducted by means of ancient Greek geometry or medieval Arabic algebra. The second stage was marked by the emergence of "function" as a fundamental mathematical concept that dominated group and set theories no less than differential and integral calculus. This stage also produced powerful efforts to eliminate the monopolistic position of mathematical analysis (integral and differential calculus) in the treatment of functions [Khinchin 1977, 62–63]. The third stage began in the mid-20th century. Its major distinguishing features were the vast expansion of mathematical tools and of the domains of mathematical competence and growing interest in "structural unity" as a universal attribute of natural phenomena. Although structural unity attracted the attention of mathematicians in the 19th century, it was not until the middle decades of the 20th century that it became the central theme of mathematical exploration. Soviet scholars showed an inclination to credit Lobachevskii with pioneering work in making the first solid and consistent step in setting the stage for the emergence of a structural orientation in mathematics [Katsiveli 1975, 20]. By creating a rich assortment of "structures" mathematics was able to explain not only quantitative but also qualitative aspects of nature and society [Bazhenov & Nutsubidze 1976, 230–231]. The structural orientation in mathematics gave scientific studies a firmer base and a broader scope of operation.

There was no general agreement on the structural orientation. Some interpreters assumed that it represented a substitute for Engels's definition of mathematics as a science of "quantitative relations" and "spatial forms," though they refrained from open criticism [Gnedenko 1972, Ershov 1970]. At the same time, some scholars viewed the two orientations as equal partners supporting each other. There was also a dwindling group of skeptics who refused to accept any part of the structuralist orientation. One such critic claimed that "mathematical structures or algebraic categories can never be objects of mathematics"; they were, he thought, "symbols without content" [Nyasanbaev 1982, 98]. He found no good reason for rejecting Engels's definition.

The recognition of "structure" as a major object of mathematical studies automatically implied an acceptance of the axiomatic method as the basic path to new knowledge and its integration into new or existing theorerical systems. This step marked a major digression from the empiricist positions in Marxist epistemology. N. Bourbaki's essay "The Architecture of Mathematics," offering a synopsis of structuralist approaches, was published in a Russian translation in a book aimed at reaching the general public [Bourbaki 1972]. In general, however, the notion of mathematical structure precipitated more philosophical than scientific discussion.

Three strong modern developments gave vivid expression to the quintessential features of the new thinking in mathematics: the triumph of cybernetics as an umbrella for a series of new branches of mathematics, the vast expansion in the operational range of mathematical logic, and the intensif ed interest in the foundations of mathematics. Each of these developments warrants closer scrutiny.

CYBERNETICS: MATHEMATICAL IMPLICATIONS

Cybernetics was generally known as a mathematical study of the processes of control and communication in self-organizing natural and social systems; it deals with "the reception, transmission and processing of messages in complex, dynamic systems, whether they be technological systems, animals, or social orders" [Kirschenmann 1970, 2]. Soviet commentators saw cybernetics as a scientif c method and a system of thought that was both a product of mathematics and a contribution to new mathematical ideas [Glushkov & Moroz 1985, 18]. The attitude of the Soviet scholarly community toward cybernetics passed through three distinct phases [Susiluoto 1982, Chaps. 9–10; Graham 1972, Chap. 11; Holloway 1974].

The f rst phase was dominated by the wave of anticosmopolitanism, particularly potent during the late 1940s and early 1950s. Heightened intellectual anti-Westernism created an atmosphere in which cybernetics could be viewed only as a system of suspect ideas. Although Wiener did not ally cybernetics with an "idealistic" philosophy, he was decidedly and clearly closer to a neopositivist orientation with a strong linguistic bias than to a materialistic philosophy. Cybernetics also gave additional support to mathematical logic at a time when there was a pronounced tendency in the Soviet Union to view this discipline as too remote from the practical needs of the day and too close to the more belligerent orientations in idealistic philosophy. The journal *Questions of Philosophy* went so far as to attack cybernetics as a "pseudo-science" and a tool of Western imperialism. It stated: "The theory of cybernetics, trying to employ the most modern computers in the study of natural and social phenomena without taking into account their unique features, is mechanicism transformed into idealism" [Materialist 1953, 216; Vykhovskii 1952, 125–127]. What work was done during this phase to prepare the ground for the acceptance of cybernetics came from mathematicians, such as A. Ia. Khinchin, who worked on a mathematical theory of information, and was protected by the relative isolation of their discipline.

During the second phase, from the mid-1950s to the early 1960s, cybernetics—aided by a concerted national effort to undo the damages of Stalinist anticosmoplitanism—rapidly found a congenial home in the Soviet Union. Indeed, as Loren R. Graham has noted, "one can f nd no other moment in Soviet history when a particular development in science caught the imagination of writers to the degree to which cybernetics did" [Graham 1972, 329]. Because a group of prominent scientists—A. N. Kolmogorov, A. Ia. Khinchin, S. L. Sobolev, and A. A. Markov in mathematics, I. I. Shmal'gauzen in biological theory, P. K. Anokhin in neurophysiology, and N. A. Bernshtein in psychology—became its ardent supporters, it found it much easier to become an academically respectable and appealing discipline. All this, however, did not mean that, particularly at the beginning of this phase, there were no reservations about the new theory and its creator. Some of the leading scientists, attuned to philosophical orthodoxy, argued that care should be taken to prevent mathematical methods from moving into substantive areas of research for which they were not suitable. They were ready to oppose scholars who treated cybernetics as an inexhaustible source of suggestive ideas for a total mathematization of science.

Some scholars voiced dissatisfaction with Wiener's choice of mathematical disciplines on which cybernetics was built; they thought that in order to attend to the distinctive properties of their research areas, individual sciences should be encouraged to rely on mathematical methods most suited to their special needs. All these criticisms were not compelling enough to prevent cybernetics from becoming one of the most exciting developments in post-Stalinist science. Wiener did not fare as well as cybernetics. Just to keep the ideological issues alive, philosophers continued to bring up selected aspects of Wiener's world outlook for unmitigated attacks. They noted, for example, that his discovery of the laws of feedback as an organizational principle of nature—the principle of negentropy—did not prevent Wiener from voicing unrealistic and pessimistic claims of the theory of "thermal death" (based on the law of entropy) [Arab-Ogly 1959, 129].

All criticisms, however, were philosophical detours, rather than parts of a fundamental and comprehensive attack on cybernetics. Philosophical skepticism quickly retreated before the grand promises of cybernetics. S. P. Sobolev and A. A. Liapunov heralded the full acceptance of cybernetics—the beginning of a new era in Soviet science:

"Cybernetics occupies a specif c place in the family of sciences. It interacts with other sciences—in some cases employing their achievements and in others helping these sciences to open new areas of research. Interrelations of cybernetics with the other sciences of human activity are generally not different from those of other disciplines. However, the striking novelty of the basic problematics of cybernetics, coupled with inadequate knowledge in our scientif c and technological circles, has produced many complications. In particular, the absence of ample literature on the problems of cybernetics in our country has created a regrettable situation. It has been most annoying that until recently an overwhelming majority of our philosophers have been ignorant of the ideas unveiled by cybernetics. It seems to us that the development of cybernetics under socialism is especially important, for its basic task is to substitute free, creative work for heavy physical labor. Cybernetics is called upon to liberate the human brain form the burdensome and exhausting work demanded in our age. In a socialist country, it offers a boundless view of perspectives for controlling various branches of the national economy, industry and planning" [Sobolev & Liapunov 1959, 259–260; Mikulak 1965; Graham 1972, 324–354].

In the third phase, during the 1960s and 1970s, Wiener's philosophical views evoked only sporadic and ineffective comments. Lodged in the substantive core of science, cybernetics built a universe of ideas where the boundaries separating scientif c from idealized worlds were not clearly def ned. Its philosophical moorings were no longer questioned: indeed, it was viewed as new corroboration of the dialectical view of nature. By establishing the "objective" character of information processes, it constructed a concrete bridge between the cosmic principles of the material world and universal development [Sobolev & Liapunov 1959, 259; Sobolev, Kitov, & Liapunov 1955, 77]. It provided considerable stimulus for Soviet scientists to match the traditionally dominant historicist orientation in the study of nature with an equally strong interest in the structuralist orientation. Structuralist approaches of Western molecular biology and quantum chemistry, psychology and linguistics, anthropology and sociology found a new home in Soviet scientif c thought, opening the doors to various systems approaches in the universe of scientif c inquiry. Cybernetics became off cially recognized as the prime mover in the inauguration of the age of computers, the backbone of the scientif c and technological revolution in the 20th century.

Cybernetics was not a specialized discipline. It was a grand theoretical orientation—a unique "style of scientif c theory." It was a synthetic orientation grounded in the world of mathematics. Its major contribution to Soviet scientif c thought was in creating a new intellectual atmosphere which helped to eliminate, or radically attenuate, the bias against the unlimited mathematization of science built into Leninist thought. For years, Marxist philosophers warned mathematicians that their science, by virtue of the highly abstract nature of its general ideas, was easy prey to all kinds of idealistic philosophies. They were cautioned to be especially vigilant in warding off the constant streams of idealistic aberrations. Particularly in the age of Stalin, Soviet mathematicians were constantly reminded of Lenin's warning that their science, in its modern version, was seriously threatened by a revival of Kant's dictum that "reason prescribes the laws of nature," reducing mathematics to equations not related to "matter" [Lenin 1970, 218].

In the post-Stalinist atmosphere, philosophers chose to abandon Lenin's warning, replacing it with Marx's forthright statement that the perfection of all branches of science was measured by the degree of their reliance on mathematics; the greater the degree of this reliance the greater the level of perfection of individual sciences [Lenin 1970, 296; *Vospominaniia o Markse i Engel'se* 1956, 66]. Especially in the eyes of post-Stalinist philosophers, cybernetics was a most telling index of revolutionary advances in the precision and scientif c authority of mathematics.

Soviet studies in cybernetics covered a vast area ranging from formal logic to epistemology, and from modern developments in mathematics to a new look at medicine and the humanities. Soviet publications in cybernetics attracted world attention; according to a list compiled by L. R. Kerschner in 1964, 249. Soviet studies in the f eld were translated into foreign languages, mostly into English [Kerschner 1964]. As Western interest in cybernetics ebbed in the second half of the 1960s translations were reduced to a trickle. Despite their proliferation, translations made little impact on scientif c thought outside Russia.

Spurred by cybernetics, mathematics underwent rapid expansion of its operation; it both penetrated all the natural sciences and made deep inroads into the social sciences. Recently considered a disreputable discipline, sociology was now a legitimate science in search of guidance from mathematics. Inf uencial voices were also heard in favor of establishing enclaves in philosophical thought open to mathematical scrutiny. The needs of modern research perspectives helped create new domains of mathematical thought.

The *Great Soviet Encyclopedia* [3rd ed. 1974]—in the article on mathematics, written by A. N. Kolmogorov—fully endorsed Engels's def nition of mathematics as a science of "quantitative relations" and "space forms" in the "real world," that is, in the world that had a traceable empirical base. In the 1960s, impressed with Bourbaki's analysis of the "architecture" of modern mathematics and encouraged by cybernetics, Soviet mathematicians and philosophers were ready to admit that Engels's def nition no longer described the full nature of mathematics: it did not account for 20th-century changes that made mathematics a study of various kinds of "abstract structures"—with no traceable roots in an empirical substratum [Bourbaki 1948, 43–45; Ruzavin 1983a, 474–477; Ruzavin 1983b, Chap. 7]. According to G. I. Ruzavin [1983a, 474], "While earlier mathematics studied mainly quantitative relations and forms of space, now it analyzes abstract structures of various types." V. S. Luk'ianets [1974, 125–126] claimed that modern physics was built by "qualitative mathematics" which he identif ed as mathematics of conceptual systems—such as "general set theory" and "theory of categories and functions." Twentieth-century advances in mathematics showed conclusively that "classical mathematics," reaching its highpoint of development in the 19th century, was only one of many possible modes of mathematical thought. Lobachevskii's and Riemann's non-Euclidean geometry showed that mathematics could advance not only by improving established principles but also by opening completely new tracts of thought. Soviet mathematicians and philosophers were also predisposed to accept Einstein's claim that mathematics was basically a "pure" product of the human mind elevated above the limits of experience.

The leading representatives of the new generation of Soviet scholars claimed that the rising levels of abstraction—and the increasing remoteness from an empirical base—helped mathematics establish "deeper and more complex" relations with the external world. By detaching itself from empirical roots, "modern mathematics, in comparison with classical mathematics, has created a more favorable condition for giving the mathematization of science more depth and wider scope." "One of the characteristic features of modern science," we were now told, "is its increasing reliance on mathematical tools which are products of the internal logic in the development of mathematics that originally appeared to have only an intramathematical value" [Stastishin 1983, 374–375]. Mathematical logic, once limited exclusively to theoretical considerations, was now a major contributor to the engineering of electronic computers [Sobolev, Kitov, & Liapunov 1955, 139].

Cybernetics became the lifeline for newly founded mathematical linguistics, bionics, and semiotics. Exploratory studies were undertaken in applying its basic theoretical categories and explanatory principles to special studies in neurophysiology, biological evolution, geology, medicine, law, and above everything else, complex technological processes in industry. It engaged the attention of A. N. Kolmogorov, the country's leading mathematician, who contributed noted studies on the theory of algorithms and automatic systems. Unlimited diversity in the applications of cybernetics was considered an aff rmation of the Marxist principle of "the material unity of the world"; Soviet cybernetics operated on the principle that there was a common mathematical approach to the technical operations of industry and the mental processes of the human mind. Cybernetics was def ned as an application of mathematical methods to the study of a domain of objective laws of nature that involved new sources and analysis of information [Berg 1964, 5–26]. In fact, the basic contribution of cybernetics was seen as opening wide perspectives for the mathematization of science.

The Marxist philosophers of the Stalin era were unanimous in viewing set theory, topology, mathematical logic, and functional analysis as disciplines whose abstruseness earned them a low social rating—a retaliation for their relative isolation from the f ve-year plans of the national economy, that is, from the immediate needs of the "real world." In the post-Stalin era, the exact opposite social valuation of mathematics became preeminent: it was fully recognized that the most advanced and broadest application came from the branches of mathematics heavily preoccupied with the problems of the internal logic of their growth. These branches formed the frontline of the structural orientation Bourbaki had introduced.

Soviet mathematicians considered the invention and mass production of digital computers the most important step in the mathematical revolution of the 20th century. Computers not only made it possible to handle vast amounts of data but played a major role in laying the groundwork for the emergence and advancement of new branches of mathematics. They became the basic instruments of artif cial intelligence, a meeting point for new branches of logic and mathematics. Technology in the precomputer age contributed to the extension of the power of the human hand; the computer-based technology of the new age strengthened the capacity of human cognition.

Computers became symbols of the unity of the most modern and most abstract branches of mathematics and the most challenging products and instruments of modern technology. Mathematicians now emphasized the role of algorithms, which improved with the help of computers; algorithms, in turn, became tools of mathematical experiments and a basis for mathematical models, an efficient substitute for more expensive mechanical models.

To a majority of Soviet philosophers and sociologists of science, computers represented the most potent and most sublime artifacts of the ongoing revolutión in mathematics. Computers and the engagement of mathematicians in the work on artificial intelligence produced a major reversal in the philosophical thought of Marxism—the acceptance of the previously criticized neopositivist emphasis on language as the true instrument of science in general and of mathematics in particular. Marxist philosophers found themselves in full agreement with Bohr's statement that mathematics was not only knowledge but also language. The exploration of formal links between grammatical rules and mathematical symbolism became the lifeline of the new mathematical logic.

In the West, various branches of mathematical knowledge stimulated by cybernetics—such as the general theory of information, linear programming, and artificial intelligence—grew into independent systems of scientific propositions. As a result of this growth, the name of cybernetics rapidly faded and eventually ceased to be referred to. In the Soviet Union, the opposite situation prevailed: the cybernetization of a rapidly growing spectrum of disciplines proceeded at full speed. The chief articulators of cybernetics were elevated to the highest rungs of academic prestige. It became a state science, viewed not only as the most fertile contribution to positive knowledge but also as an infallible source of guideposts for the consolidation of socialism and communism [Novik 1990, 12]. The more politicized cybernetics became, however, the more it drifted from realistic goals and, as I. B. Novik [1990, 14] has intimated, it became a wheel of state ideology and to a certain degree a pseudoscience.

Cybernetics was supported by Soviet authorities more for political than for scientific reasons. Communist leaders saw in cybernetics a major tool for enhancing the multiple system of social control by automating it. Soviet experts in cybernetics worked assiduously and successfully on convincing the political authorities about the limitless potentials of applying a "bourgeois science" to achieve communist goals. The government was interested less in a future communist utopia of full social equality than in the immediate need to strengthen the existing system of total control over the structural component of society—a system that showed increasing signs of deterioration.

Despite its ideological involvement, cybernetics deserved much credit for having encouraged studies on the role of internal logic in the construction of mathematical knowledge and for having broadened the links between mathematics, the expanded range of science and modern technology. It stimulated special interest in mathematical logic and the foundations of mathematics and brought Marxist epistemology a few solid steps closer to Western philosophical thought, especially recognizing the tangible role of the subjective factor in the construction of the true picture of the world. Cybernetics also encouraged the search for the axiomatic method in mathematics, which was fully rejected by Stalinist philosophers.

E. Kol'man's continued war on "mathematical idealism," however, showed that traces of Stalinist philosophical orthodoxy was still present [Kol'man 1965, 208–210].

It was not until the 1980s that, under Western influence, cybernetics began its readily discernible decline in vigor and prestige in the world of science. At first, it ceased to be viewed as a unique style of general scientific thought and was seen as a component of general systems theory, concerned exclusively with control systems in technology, biological phenomena, and social organizations. Then, it rapidly ceased to be referred to as a distinct body of general theory, its full demise taking place on the eve of *perestroika*. At the time of its demise it was known more as an integral part of state ideology than as a unique and promising style of scientific inquiry.

The last Soviet book on cybernetics, a collection of essays written mostly by persons occupying high positions in the USSR Academy of Sciences, was published in 1984, at a time when unmistakable signs indicated that Wiener's creation was losing ground as a serious scientific challenge. Looking over its relatively short historical span as a major national preoccupation, A. N. Kolmogorov, the nation's most prominent mathematician, concluded that Soviet studies in cybernetics followed a course of extremes, which led them to produce "a good deal of exaggeration, on the one hand, and oversimplifications, on the other" [Kolmogorov 1989, 34]. However, Kolmogorov [1987] was ready to recognize the major role cybernetics had played in providing the stimulus for taking mathematics to new heights of achievement and previously unexplored domains of inquiry. Spurred by the promises of cybernetics, he made significant contributions to mathematical theories of information and algorithms.

MATHEMATICAL LOGIC

Not until the end of the Stalin era did philosophers recognize the growing role of mathematical logic in the family of modern sciences. The journal *Questions of Philosophy* noted in 1950 that the development of mathematical logic was closely linked with the critical issues generated by the rapid growth of mathematical theories during the preceding hundred years. The set-theoretical "paradoxes" added a more acute sense of urgency to the need for a special branch of science concerned with the logical foundations of mathematics [Tugarinov & Maistrov 1950, 335]. Wary of its "idealistic" foundations, Stalinist philosophers claimed that Marxists could tolerate only a mathematical logic that incorporated theories and axioms based on "criteria of truth" anchored in "objective reality"—in "practice." In their opinion, to deal exclusively with "pure forms" meant to succumb to "subjectivism," a digression from both Marxism and science [Tugarinov & Maistrov 1950, 333]. They offered no concrete plans for combining the study of "pure forms" with the study of practical criteria of truth and were determined to treat mathematical logic only as an auxiliary branch of scientific knowledge.

In general, Marxist critics viewed mathematical logic as a science especially vulnerable to the corruptive influences of "idealistic" philosophies represented by logical positivism. No Stalinist philosopher took exception to S. A. Ianovskaia's claim that mathematical logic was one of the less developed branches of mathematics in the Soviet Union [Ianovskaia 1948, 45]. A small residual group of Marxist philosophers reasoned that mathematical logic was essentially incompatible with dialectical materialism [Shvyrev 1983, Vykhovskii 1952]. Without exception, they were motivated by ideological reasons.

Heavy criticism of the philosophical foundations and ideological implication of Western mathematical logic did not prevent the publication of Russian translations of *The Foundations of Theoretical Logic* by D. Hilbert and W. Ackermann and of *Introduction to the Logic and Methodology of Deductive Sciences* by A. Tarski in 1947–48. The *Questions of Philosophy*, the leading journal devoted to Marxist theory, considered these books open expressions of the philosophical views of logical positivism and lamented their Russian translation in the first place [Tugarinov & Maistrov 1950, 339–342]. It went as far as to accuse Ianovskaia, the author of introductions to the Russian translations of both books, of expressing a conciliatory attitude toward idealistic philosophy. Ianovskaia apologized for philosophical indiscretions and agreed that an error was made in publishing Hilbert's and Ackermann's book in a philosophical, rather than in a mathematical series. Inadvertedly, she violated the Stalinist rule of the inseparability of science and Marxist philosophy [Ianovskaia 1951, 342].

The attitude toward mathematical logic underwent rapid and profound change soon after Stalin's death. This development was intensified by the recognition of cybernetics as a legitimate branch of scientific knowledge and as a potent addition to modern scientific methodology. Mathematical logic became the theoretical and methodological base of a new mathematics that ushered in the age of electronic computers [Semenov & Uspenskii, 1986]. In 1966, Moscow University established a permanent academic position for mathematical logic and soon international journals and symposia in the field took serious note of Soviet contributions to the field. In the same year, the journal *Priroda* (*Nature*) described the new situation:

"Not so long ago . . . mathematical logic was among the most abstract branches of science that were as far removed from practical problems as earth was from the sun. Today the situation has changed. Space conquests have brought the skies closer to the earth and have made mathematical logic one of the most practical sciences. As the theoretical base of the full range of modern technology of programming, it is the power behind the miraculous work of electronic computers which play chess, translate from one language to another, control technological processes, and solve scientific riddles. It is not exceptional that an abstract science, at first far removed from practical application, suddenly becomes one of the most practical sciences" [Boltianskii & Ryvkin 1966, 49–50].

Soviet historians of science now argued that efforts to combine mathematics and logic had deep roots in the Russian intellectual tradition. Lobachevskii, in particular, received plaudits for having been one of the progenitors of modern logical methods in mathematics; after all, he adduced a tight system of logical arguments showing the unprovability of Euclid's fifth postulate. Jean van Heijenroot, a Dutch scholar, gave Kolmogorov credit for having presented "the first systematic study of intuitionist logic." [From Frege to Gödel 1979].

Interest in mathematical logic was two-pronged: on the one hand, it concentrated on expanding and refining the use of logic in the clarification of critical issues in the foundations of mathematics and in bolstering the rules of mathematical proofs, and, on the other hand, it made logic more effective and better adapted to the needs of science by relying on mathematical methods.

The consolidation of mathematical logic as a powerful body of formal principles and rules came at a time of accelerated growth of new logics—such as modal logic, many-valued logic, and projective logic—some tied to mathematics and some to other systems of knowledge. These logics served as a bridge between mathematical logic as a fully developed branch of mathematics (and the sparkplug of cybernetics) and as a distinct and clearly formed branch of philosophy. As such, mathematical logic was ideally positioned to serve as a testing ground for meeting the pressing need to accommodate dialectical materialism to the rising complexity of scientif c thought.

As a branch of philosophy, mathematical logic had a special task: it bolstered the scholarly community in its effort to create an enclave in philosophy that would enjoy relative detachment from ideological concerns and would assist philosophy in facing the fundamental questions of the formal structure of scientif c cognition and logic. The nature and the scope of the new challenge were clearly stated by A. A. Zinov'ev, an expert in modern logic as a link between philosophy and mathematics:

"Under the inf uence of the mathematization of sciences and the successes of mathematical logic in the last few decades, a special branch of logico-philosophical research has developed. Its essence is the use of the ideas, the apparatuses (calculi) and methods of mathematical logic and mathematics (exact methods) in the solution of a series of traditional problems of formal logic and philosophy as well as of new problems of the methodology of science specif cally connected with the development of contemporary science. In this branch one considers the epistemological interpretation of formal systems of logic, constructs formal systems for the express purpose of describing various aspects of human cognitive activity, solves certain problems of philosophy by means of logico-mathematical constructions, and uses the accomplishments of logic to overcome philosophical diff culties in the natural sciences" [Zinoviev 1978, 92].

Zinov'ev placed the philosophical study of mathematics and the mathematical study of philosophy in a complementary relationship and in this complementarity he saw one of the major developments in both modern science and modern philosophy. He acknowledged the broad opposition to the mathematization of philosophy, but he also noted the increased application of "exact methods" to the problems traditionally classif ed as philosophical. He viewed mathematical logic as a generic term for two major disciplines—the one f rmly lodged in the general system of modern mathematics and the second operating within the rapidly expanding realm of modern philosophy. Zinov'ev, however, made clear that just as logic cannot hold an exclusive right on mathematical constructions so mathematics cannot be the sole master of logic and philosophy. By "exact methods" as methods of new philosophy he meant the methods of logic, not all of which are allied with mathematics. As part of "exact methods," mathematics, in his opinion, contributed to the study of philosophical structures from positions unencumbered by intellectual—and ideological—biases. These were not mere programmatic declarations, but potent ideas meticulously and extensively elaborated in numerous logical studies published by Zinov'ev and his colleages.

The philosophers of mathematics focused their attention on symbolic logic and mathematics as effective mediators in the complex relations between mathematics and philosophy as well as between mathematics and technology [Abdil'din *et al.* 1985, 334]. Their avowed reliance on dialectical materialism as a "general philosophy" of mathematics did not prevent them from recognizing the rights of mathematicians to select, formulate, and examine the fundamental philosophical questions of their individual disciplines. The growing emphasis on the need for advancing and enlarging the corps of mathematicians actively engaged in philosophical intricacies of their science, however, did not produce the expected results. During the 1970s–1980s, two developments took place: a decline of interest by mathematicians in philosophical questions and the rapid growth of philosophers of mathematics who made their reputations as philosophers rather than as mathematicians. While the number of philosophers increased considerably, the number of mathematicians interested in philosophy notably declined.

In the early 1960s, Marxist theorists began to concentrate on building a new branch of philosophy which they labeled dialectical logic. A subject of many studies, the new discipline represented a special accommodation of Marxist thought to the world of ideas released by mathematical logic. Although dialectical logic had no clearly def ned subject of inquiry, it usually focused on the basic categories of cognitive processes underlying the the formation of general abstractions [see for instance the article "Logika dialekticheskaia" in the *Filosofskaia entsiklopediia*, Vol. 3, pp. 209–221]. While fully recognizing the contributions and aspirations of mathematical logic—and of all new varieties of logic—it was guided by the idea that there was a need for a logic that could also consider the substance of sociocultural inf uence. It was based on the premise that no formal logic could fully explain the intricacies of human cognition [Subbotin 1987, 146]. Calculus was seen as a product not only of internal logic but also of external inf uences.

Dialectical logic neither enriched the Marxist philosophy of science nor made an impact on the substance and development of mathematical logic. Its involved discussions were of a scholastic nature [Abdil'din *et al.* 1985, Gorskii *et al.* 1978, Konstantinov *et al.* 1966]. Not only mathematicians but also most philosophers of mathematics consistently avoided references to its massive, hyperbolic, and generally nonproductive effort to strengthen dialectical materialism. It showed increasing signs of the dwindling fortunes of orthodox positions in the Marxist philosophy of science. The blossoming of mathematical logic came on the wings of pronounced philosophical digression from the traditional principles of Marxist thought.

Solid advances in mathematical logic led to the well established proposition that while mathematics was essentially a product of logic, modern logic owed its primary debt to mathematics [Barabashev, Demidov, & Panov 1986, 151]. Explanation of the interdependence of mathematics and logic gave only a secondary—and rather unelaborated—position to the Marxist version of materialistic dialectics. Relieved of much ideological pressure, mathematical logic became a perceptible tributary to one of the mainstreams of scientif c thought.

There was a traditional tendency, however, to attribute a distinct national characteristic to Soviet mathematics in general. This tendency was clearly recorded by A. P. Iushkevich, an internationally acclaimed historian of mathematics, who made a comparision of Soviet and French mathematics. He wrote that in the Soviet Union mathematics was treated not only on the level of internal impulses for development but also on the level of interaction with such external conditions as technical needs and cultural values. In France, by contrast, he thought that mathematics was treated almost exclusively as an internally stimulated activity [Iushkevich 1987, 39].

The line separating logic as a science from logic as a part of philosophy was traditionally blurred and uncertain. The rise of mathematical logic led some Soviet scholars to conclude that the time had come to abandon logic as a philosophical discipline. Modern logic, they reasoned, was in a position to solve its own philosophical problems without depending on outside help. They thought that “philosophical logic” was open to nonscientif c methods of inquiry [Smirnov & Tavanets 1974, 25]. Their obvious intention was to achieve a full separation of logic from dialectical materialism, a process that was already taking place.

THE FOUNDATIONS OF MATHEMATICS

The third major development during the 1960s–1980s was directly related to a resolute concern with the foundations of mathematics—a search for deeper meanings of logical, epistemological and ontological aspects of mathematical knowledge. Interest in the foundations of mathematics grew in scope and intensity after Hilbert—in a paper read at the Second International Congress of Mathematicians in Paris in 1900—formulated 23 mathematical problems waiting for adequate solutions. In the Stalin era, Soviet philosophers had rejected all Western orientations in the foundations of mathematics. Rejections were always stated in philosophical terms; all orientations were treated as victims of concessions to idealistic philosophy: logicism to “objective idealism,” formalism to Kantian apriorism, and intuitionism to “subjective idealism” [Mikhailov 1965, 62]. The Marxists were guided by the principle that a Soviet orientation in the foundations of mathematics could be built only on the total ruins of Western thought. Not all Soviet mathematicians agreed with this view. Kolmogorov’s and Khinchin’s early efforts to advance a unique elaboration of intuitionism by detaching it from obvious references to idealistic philosophy were too subtle and too intricate to attract the attention of the defenders of Marxist orthodoxy [Khinchin 1926, Kolmogorov 1979]. Marxist scholars concentrated more on exposing the “weaknesses” of the Western foundations of mathematics than on building a general theory of their own.

During the 1960s, the situation changed dramatically. It was now recognized, sometimes quite forcefully, that a Marxist orientation can only be built by accepting the positive contributions of dominant Western views. As one writer put it, an inquiry into the “dialectical interaction of various approaches to the foundations of mathematics” was the safest path to a general theory acceptable to Marxist scholars [Mikhailov 1965, 62]. This did not mean that a group of traditionalists in the philosophy of mathematics had relinquished their exclusive concern with the weaknesses of “idealistic” orientations. It meant that the strongest and most creative trend in mathematics favored a discontinuation of the Stalinist policy of isolating Marxist philosophy from the mainstream of Western orientations in the foundations of mathematics. The architects of the new orientation admitted that it was impossible to construct a unif ed and comprehensive theory and that the time had come to view the future of the foundations of mathematics in the continuing existence and growing interdependence of diverse orientations. Now Western orientations in the foundations of mathematics were objects of carefully balanced combinations of approval and disapproval.

Formalism was unacceptable in some of its general theoretical formulations, but it was credited with irrevocable contributions to the formal analysis of both logic and “techniques of our thought.” Hilbert’s procedures for making formalization processes unencumbered by extraneous ideas coming from everyday experience or from laboratory experiments became a “universally recognized feature of mathematics” [Zhukova 1976, 1567]. Hilbert did not recognize dialectical discontinuities, or leaps, either in nature or in mathematics; this did not prevent him, however, from enriching science with “profound” theoretical insights [Protopopov 1983, 73–75]. Soviet mathematicians paid homage to Hilbert’s leading role in the systematic study of the formal structures and internal consistency of mathematical systems. Some of them were now ready to acknowledge the debt of Soviet constructivism, not only to intuitionism but also to formalism [Aleksandrov 1964, 334–335].

Logicism received its share of criticism, but it was also the recipient of positive acclaim. Bertrand Russell's "theory of types" was wholeheartedly approved as a design for resolving one of the paradoxes of set theory: the classif cation of mathematical objects. Some writers noted the vital part played by the work of Russell and Whitehead "in introducing more rigorous forms of mathematical conceptualization" [Burova 1976, 74]. Ianovskaia referred to Russell's and Whitehead' *Principia mathematica* in an effort to show that scholars identif ed with "idealistic" philosophy could advance scientif c ideas enriching dialectical materialism [Ianovskaia 1964, 229]. Frege, another stalwart of the logicist orientation, received full credit for showing conclusively that a unitary logic could no longer meet the needs either of science or of philosophy and that the time was ripe for the creation and recognition of many logics, each with a distinct set of norms, reference points, and operative principles. Some Soviet mathematicians thought that the logicist orientation was correct insofar as it provided a contextual determination of mathematical concepts, but that it was incorrect in claiming that the principles of mathematics were deductible from logical suppositions [Smirnov & Tavanets 1974, 9–10].

Intuitionism received more attention than formalism and logicism. V. F. Asmus resented the determined efforts by Brouwer and Weyl, the original architects of this orientation, to eliminate all logic, with the exception of the "intuitionist type," from the foundations of mathematics [Asmus 1965, 286]. In 1965, A. Mostowski, a leading Polish expert in the foundations of mathematics, gave a contrary interpretation: he wrote that A. A. Markov, the founder of the Soviet school of constructive mathematics, accepted the basic premises of intuitionist logic, even if he did not admit it publicly [Mostowski 1965, 105]. Subsequently, other leaders in the foundations of mathematics made the admission more direct and categorical.

In the heyday of Stalinist anticosmopolitanism, A. D. Aleksandrov, a mathematician sensitive to philosophical-ideological issues, thought that intutionism was as "reactionary" as logicism and formalism and that it should be categorically rejected. He wrote in 1951 that intuitionism was "subjective idealism in mathematics which refuses to attribute any objective meaning to mathematics" [Aleksandrov 1951, 6]. In the age of post-Stalinist thaw he took a more conciliatory position: "We may cite intuitionism which in its mathematical interpretations appears as subjective idealism. However, a more rational trend ... has led to the concern of intuitionist logic with real problems and... "constructive arguments" as concrete mathematical entities without a trace of idealism" [Aleksandrov 1970, 196].

Soviet scholars also agreed with A. Heyting's bold statement, made in 1962, that "a new form of mathematics is born, in which we know at any moment whether we work on the intuitive basis or not, which part of the work is purely formal, and which platonic [logical] assumptions to make." Nor were they now inclined to challenge Heyting's assertion, made at the same time, that "none of the conceptions of mathematics today is as clear-cut as it was in 1930" [Heyting 1962, 194–195]. According to V. S. Luk'ianets [1974, 336–337], modern mathematics has rejected every belief in the "absolute exclusiveness and unlimited authority of individual orientations in the foundations of mathematics," but it has also rejected a "blind negation" of any one of them.

For many years Soviet philosophers tried to draw a picture of mathematics as a science whose inner unity was built on firm epistemological and logical foundations. They relied on the alleged internal unity of mathematics as the most basic scientific expression of the unity of nature. During the 1960s, the idea of unity gave way to accelerated search for new methods and a constantly shifting theoretical base. Now experts went along with Bourbaki's comparison of mathematics to a typical city whose never-ending search for architectural and functional unity is impeded by the perennial rebuilding in the center and chaotic growth of suburban districts. The lack of unity was not a sign of mounting uncertainties and uncontrollable wandering of mathematical structures; it was an inevitable product of the broadly based upsurge in the search for new modes of mathematical expression and new areas of mathematicized thought. Foundational disunity was an unavoidable byproduct of the rapid ascent of mathematics to the new heights of abstraction and more complex inner structures.

Gödel's proof that axiomatic systems in mathematics, including ordinary arithmetic, contained undecidable formulae provided a new challenge to the idea of the general unity of mathematics. What Gödel emphasized was that it was impossible to achieve structural or axiomatic unity of individual branches of mathematics, let alone of mathematics as a general science. In the words of E. Nagel and J. R. Newman [1958, 6], Gödel noted that "no final systematization of many important areas of mathematics is available, and no absolutely impeccable guarantee can be given that many significant branches of mathematical thought are entirely free from internal contradictions." No Soviet philosopher or mathematician made an effort to advance arguments contrary to Gödel's thesis.

Stalinist philosophers were not inclined to explore the avenues leading to a synthesis of contradictory theories, particularly when these were directly related to philosophical issues. Instead, they preferred to dismiss or rebuff all elements of thought standing in the way of a simply conceived and categorically upheld structural unity of philosophical and logical principles of "Marxist mathematics." The philosophers of the new era, by contrast, joined the ranks of leading mathematicians in recognizing that the real strength of mathematics was in the interdependence of its contradictions rather than in the unity of its overgeneralized assumptions and ideological overtones. The dictum that "dialectics thrives on contradictions" received much attention and recognition in the new philosophy of mathematics. It marked a general rejection of Stalinist anticosmopolitanism, particularly its emphasis on the unity of Soviet science based on the primacy of materialistic thought in the Russian national tradition. If there was a promise of unity in modern mathematics, it came neither from the sociological underpinnings of the Russian scientific tradition nor from the rigorous methods of formal logic: it came from the joint contributions of different foundational orientations, each with its own formal theoretical system and style of proofs and each claiming a specific domain as its area of operation. In the words of a philosopher of mathematics: "It is obvious that no foundational orientation, in its historical essence, can be final and absolute, and that no single orientation can set up the limits of the development of mathematics" [Burova 1976, 111].

It would be misleading to assume that all Soviet commentators on leading Western orientations in the foundations of mathematics showed the same degree of moderation and tolerance. For example, the defenders of Marxist orthodoxy were still heard from, particularly among the philosophers of mathematics. One such defender was N. A. Litsis, the author of a book on “the philosophical and scientific meaning of N. I. Lobachevskii’s ideas.” Closely following the epistemological ideas of Lenin’s *Materialism and Empiriocriticism*, he viewed Hilbert’s formalism and Russell’s logicism as specific manifestations of idealistic efforts to deprive mathematics of an empirical base. He wrote that Poincaré, Mach, Klein, Hilbert, Russell, and “all other idealists-philosophers” shared a philosophical view that “denied the reality of the external world. . . .They viewed mathematics, not as science of quantitative relations and spatial forms, but as a complex of ‘definitions’ and ‘conventions,’ or identified it with the laws of formal logic” [Litsis 1976, 218]. Litsis believed only in the “objective” base of mathematical theories, in the empirical underpinnings of mathematical knowledge, and in utilitarianism as the motive force in the evolution of scientific knowledge [Litsis 1976, 206–207]. Supporters of Marxist–Leninist orthodoxy, however, were quite rare.

The recognition of specific contributions by the leading Western schools in the foundations of mathematics did not prevent Soviet scholars from advancing an orientation that was strictly their own. A. A. Markov, N. A. Shanin, and numerous colleagues worked on a distinct Soviet version of constructivism, an offshoot of the intuitionist orientation [Shanin 1958, 223–225; Mostowski 1965, 105; Panov 1994, chap. 5; Subbotin 1977, 61–63; Ruzavin 1983b, 154]. In the words of A. G. Dragalin, “constructive mathematics can be viewed as a branch of mathematics, characteristically involved in the study of constructive objects with the help of algorithmic methods” [Dragalin 1974, 59; Dragalin 1979, 642; Luk’ianets 1980, 144–151]. Following the intuitionist tradition, leaders of the Soviet constructive orientation relied mainly on methods that recognized only series with identifiable objective existence. They fully accepted the claim of A. A. Fraenkel and Y. Bar-Hillel [1958, 207] that “the fundamental thesis of intuitionism” in almost all of its aspects is that “existence in mathematics coincides with constructibility” [Markov 1964, 50]. The rule of constructibility states that no mathematical entity is recognized unless it can be presented in a finite number of steps. In the words of A. A. Markov [1968, 283], “According to the fundamental thesis of constructive mathematics, we consider in this science merely the results of our activities realizing these constructions. We admit the abstraction of potential realizability; i.e., we abstract from practical limitations of our abilities in space, time, and material and we argue as if such limitations were absent.”

P. Lorenzen, a leading German expert in the foundations of mathematics, offered a description of constructivism as a general, rather than as a unique Soviet orientation. In constructivism he saw a new effort to reinterpret and build upon established mathematical thought. Its notion of potential infinity recognized mathematics as both an open system of abstract knowledge and a cultural reality with deep and continuous roots in history. Constructivism represented an effort to overcome two philosophical dilemmas of the modern era: epistemological skepticism (which placed more emphasis on criticizing old theories than on creating new ones) and moral subjectivism (which simply ignored unfavored theoretical views) [Lorenzen 1968, 141–142].

Markov and his Soviet colleagues argued that in constructive mathematics all operations must begin with a clear identification of intuitively selected classes of constructive objects—such as letters in a specific alphabet, finite graphs, and integer matrices. They also required identification of the subclasses of these classes and their properties. With preliminary steps completed, constructive orientation concentrated on a recursive analysis of constructive processes, relying heavily on the precision of algorithms. Soviet constructive mathematics treated algorithms as the most reliable links between theoretical constructions and concrete objects of inquiry, as an airtight instrument for safeguarding the objective nature of mathematical abstractions, and as a particularly handy tool for accelerating the mathematization of scientif c knowledge [Kedrovskii 1972, 149]. With no def nition of its general method, it concentrated on the study of constructive processes in individual branches of mathematics [Dragalin 1973, 111–128; Dragalin 1974, 55–58]. One expert advanced a theory of constructive topological spaces, and another applied the Riemann integral to constructive functions.

A special effort was made to create axiomatic expression for the theory of constructive objects. The problem of real numbers and functions, or variables, occupied the central position in constructive mathematics. Soviet scholars thought that this kind of engagement helped them in transforming the foundations of mathematics from a special branch of philosophy to a clearly def ned and rapidly expanding branch of knowledge.

In building their intricate structures, Markov and his Soviet colleagues stayed close to a residual national tradition associated with intuitionism and clearly avoided more than superf cial and formal associatation with dialectical materialim. The guardians of Marxist philosophy had no argument with the constructivist effort to state all propositions in everyday language, to avoid references to actual inf nity, and to make algorithms not only the key mode of foundational operation but also a safe mechanism for keeping constructive objects free of unwanted subjective admixtures. In Brouwer's categorical rejection of Kant's notion of space and time as immanent endowments preceding every experience and total acceptance of explanations of causal links in natural processes as the full measure of scientif c progress, they saw an indisputable aff nity of intuitionism and dialectical materialism [Brouwer 1962, 693–694].

Soviet mathematicians and philosophers welcomed intuitionist recognition of the sociocultural roots of mathematical knowledge. There was no serious effort, however, to give this recognition solid empirical backing. Constructive mathematics was exclusively a product of intricate logical procedures. In his article "Constructive Mathematics" published in the *Great Soviet Encyclopedia* (3rd ed., 18, 99–100). Markov stated directly that constructive mathematics was built on constructive mathematical logic, and that the latter discipline had little in common with classical logic as applied in set theory. Constructive mathematics made only passing and expendable references to dialectical materialism. Responding more to the favorable atmosphere of the post-Stalinist thaw than to the facts of science, Marxist philosophers generated no criticism either of constructive logic or of constructive mathematics.

Markov's open admission that constructive logic provided the foundation for constructive mathematics went against the claim of Stalinist interpreters of dialectical materialism that because logic studied form rather than substance, it could not be considered part of the mainstream in mathematical thought but only an auxiliary discipline. Marxist philosophers accepted Markov's propositions not only as a national contribution to the ongoing discussions in the foundations of mathematics, but also—and mainly—as a potent source of ideas for strengthening the role of computers in the system of state controls over the national economy. Algorithm, the chief instrument of constructive mathematics, was considered the most effective method for linking mathematics with cybernetics-inspired "automatic" control systems.

The community of Soviet mathematicians did not accept the arguments of constructivists with universal enthusiasm. Appraisal ranged from carefully worded signals for caution to Ianovskaia's ebullient effort to make the scientif c achievement of the constructive approach to the foundations of mathematics equal in importance to the revolutionary sweep of Lobachevskii's pioneering work in non-Euclidean geometry [Ianovskaia 1973, 203].

Marxist philosophers continued to attack "idealistic aberrations" in Western mathematics, but they now behaved as if these aberrations were more open to reconciliation with Marxist doctrine. Viewed only recently as totally incompatible with dialectical materialism, many foundational ideas were valued as contributions to both science and philosophy. Soviet philosophers and experts in the foundations of mathematics recognized the changing conditions at home as a key factor contributing to a more open and less dogmatic attitude toward Western ideas. But they also believed that a major crisis in Western philosophy had produced changes signifying a retreat from positions Marxist philosophers had found most objectionable. For example, they thought that logical positivism, which they viewed as a static and nonhistorical orientation, was seriously challenged by the philosophical work of Thomas Kuhn, Imre Lakatos, Stephen Toulmin, and Paul Feyerabend [Kedrovskii 1972, 221]. Soviet scholars also claimed that there was a gradual drifting of Western philosophy in general and of the Western philosophy of mathematics in particular toward the positions of materialism. For example, they thought they had detected clear elements of materialism in *The Foundations of Mathematics* by E. Beth and in the general orientation of the Swiss journal *Dialectica*, with the mathematician Paul Barnays as one of its editors [Ruzavin 1968, 290].

Logical positivism, a modern heir to Ernst Mach's neopositivism, was bitterly contested by Stalinist philosophers and was treated as the most fully crystallized and most pernicious expression of idealism in epistemology. Marxist philosophers resented the systematic efforts of the followers of this orientation to limit the general theory of scientif c knowledge to logical and linguistic forms. Most philosophers in the post-Stalin era continued to criticize various aspects of logical positivism, particularly its disregard of the historical approach to knowledge. At this time, however, they were also ready to acknowledge the substantial contributions of logical positivism to such new disciplnes as cybernetics and mathematical logic, two of the most popular branches of modern learning in the immediate post-Stalinist years [Vakhtomin 1986, 143]. To neopositivism in general they gave credit for pioneering work in the application of the formal language of mathematical logic to the study of philosophical problems [Ershov 1970, 86]. The acknowledgement came at a time of heightened interest in applying the resources of mathematical logic to the study of formal aspects of Marxist philosophy.

Mathematicians, rather than philosophers, played the decisive role in selecting intuitionism as a base for a special national variant of the constructive orientation in the foundations of mathematics. Kolmogorov in 1925 and Khinchin in 1926 helped to lay the foundations for quiet but sustained interest of Soviet mathematicians in exploring the potentialities of intuitionism [Kolmogorov 1979; khinchin 1926]. Soviet philosophers, in contrast, waged a sustained war on intuitionism until the mid-1950s. In the Stalinist period, they looked at intuitionism as a special expression of "subjective idealism," directly opposed to the "objective" moorings of Marxist epistemology.

In a way, the Soviet version of constructivism was a unique synthesis of the “idealistic” principles of intuitionism and the “materialism” of Marxist thought. As such, it combined intuition as the initial step in constructing mathematical objects with primary emphasis on algorithms as safe instruments for limiting the effects of intuition to computable functions. In a deliberate effort to prevent drastic digressions from Marxist theory, representatives of the Soviet constructivist orientation rejected any idea of “basic intuition.”

In the West, intuitionism was generally viewed as an orientation far removed from Marxist ideas. Heyting [1964, 42] relied on intuitionist positions when he called mathematics “a production of the human mind” and “a free, vital activity of thought,” a view diametrically opposed to the epistemological positions built into Lenin’s *Materialism and Empiriocriticism*. To Lenin, mathematical knowledge was neither “a production of the human mind” nor a “free” activity, but a specif c ref ection of the external world existing independent of the human mind. Lorenzen [1968, 140] called intuitionism a “golden midpoint between two extremes in the foundations of mathematics: the formalist orientation that had no room for the notion of inf nity and the Cantorian orientation which recognized “an inf nity of inf nities.”

Soviet scholars relied on a variety of arguments favoring a conciliatory attitude toward intuitionism. V. F. Asmus, for example, claimed that although Brouwer did not give a direct explanation of what he meant by “intuition,” he def nitely did not identify it with supersensory or transrational phenomena [Asmus 1965, 267]. M. I. Panov, a leading Russian interpreter of intuitionism, was categorical in both rejecting the views emphasizing incompatibility of intuitionism and constructivism and in claiming that one of the basic similarities between the two orientations was in recognizing the vital role of “social processes” in the growth of mathematical knowledge [Panov 1988, 54–59]. He also claimed that Bouwer’s orientation was dominated by equal emphasis on “intuition” and “constructibility.”

The constructivist orientation did not receive universal support in the community of Soviet mathematicians. A small and unheralded group of mathematicians represented by A. S. Karmin, Ia. A. Petrov, and G. G. Shliakhin adhered strctly to the principles of formalism. Mathematics, they argued, is exclusively a formal language. Mathematicians idealize reality and make their abstractions part of formal systems. The incontrovertibility of these abstractions is the *sine qua non* of mathematics. When incontrovertible systems of abstractions are applied to empirical reality, they become physics, chemistry, biology, or any other science. Conforming strictly to the rules of logic, mathematics has no ontological base [Voitsekhovich 1984, 114]. Since mathematics is assumed to deal only with form and not with substance, it stands above the Marxist—or any other—rules of social determinism. Petrov and his group did not deny Marxist sociology of knowledge; they only sought, by implication, to remove it from the foundations of mathematics. For obvious reasons, the formalist orientation had only limited support in both mathematical and philosophical circles. While the constructive orientation gave social determinism a nominal and ineffectual reception, Soviet “formalists” disregarded it completely.

CONCLUSION

From the very beginning of the Soviet era, the leading mathematicians sought to avoid direct conf ict with Marxist philosophy. They either ignored dialectical materialism altogether or reduced it to generalities that did not hinder the normal growth of their discipline.

In times of crisis, they made tenuous and transitory concessions to varied pressures coming from ideological sources. Under their inf uence, dialectical materialism abandoned some of the more rigid tenets of Leninist epistemology. The majestic remoteness of mathematics from the hot spots of ideological pressure and the strength of its roots in national tradition were the prime factors helping Soviet mathematicians to stay with—and enrich—the constant stream of new thought in the multiple branches of their discipline.

The post-Stalinist era produced long lines of philosophers working in the realm of mathematics, but these philosophers showed little similarity to their Stalinist predecessors. Instead of searching for—and criticizing—philosophical and ideological impurities in the professional work of Soviet mathematicians, they concentrated on preparing popular accounts and integrated overviews of philosophical subtleties in the ongoing foundational debate and in the arterial avenues of mathematical thought. They were especially interested in analyzing epistemological, methodological, sociological and scientif c results of the ongoing eff oresecence of mathematical thought. Contributions of modern mathematics to the science-anchored world outlook played a preeminent role in philosophical discussion. Philosophers did not abandon dialectical materialism; they merely accommodated it to developments in "new" mathematics. They succeeded in lessening the negative effects of dialectical materialism as a state philosophy on the growth of scientif c thought. They also made dialectical materialism a f uid and eclectic method of philosophical discourse.

Stalinist philosophers viewed scientif c knowledge exclusively as a response to socially generated needs. In the post-Stalinist era, this view was considered only partially true. Mathematical knowledge, for example, was now viewed as a result of both external causes and the logic of internally generated stimuli [Morozov, Petrov, & Perminov 1977, 77–78]. Differential calculus was seen both as a mathematical tool designed to help mechanics to solve some of its strategic problems and as a system of abstractions created by the stimulus of internal development [Morozov, Petrov, & Perminov 1977, 125]. Moreover, the philosophers of science thought that the most signif cant advances in mathematics after the early decades of the 19th century came from the immanent dynamics of mathematical cognition. It was generally conceded, for example, that Lobachevskii's non-Euclidean geometry did not come from an effort to solve a practical problem of social origin but was a result of "a natural striving to bring to light all topically possible geometrical systems—a striving to unravel the structure of the geometrical space around us" [van Heijenoort 1979, 156–157].

In the Stalin era, Marxist philosophers harassed mathematicians whose scientif c views, in their opinion, did not conform to Marxist theory. The Moscow school of mathematics, lodged in set theory, was the target of vicious attacks, because its leaders—particularly Egorov and Luzin—were accused of harboring scientif c views tinged by philosophical idealism. In the post-Stalin era, philosophers lost their right to criticize Soviet scientists either on scientif c or on philosophical grounds. The public harassment of scientists by philosophers was abandoned. The new rule made the scientif c community sole authority for the certif cation of scientif c knowledge. Although criticism of Western philosophical views did not cease, it became more temperate and more precise in keeping separate accounts of scientif c and philosophical arguments. Much more competent in both philosophy and mathematics than their Stalinist predecessors, Soviet philosophers now worked more toward broadening the compass and general appeal of mathematical culture than toward exposing ideological enemies.

In the Stalin age, Soviet philosophers were interested primarily in interpreting the high theory of modern science in the light of the guiding principles of Marxist theory. In the post-Stalinist era, the task of philosophers underwent a major transformation: they now concentrated primarily on adapting Marxist theory to the spirit and theoretical substance of science. In mathematical logic and the foundations of mathematics, Soviet scholars—both philosophers and scientists—spent as much time in absorbing non-Marxist developments in the West as in articulating home-grown Marxist positions.

Marxist scholars continued to cultivate, and to expand, the realm of "dialectical logic," a broad study of the Marxist views on the principles of logic. This discipline, diffuse and vague, was in no way intended to compete with the vigorous developments in logic grounded in mathematics and in proliferating systems of non-Aristotelian symbolism. Primarily as a branch of Marxist philosophy, its task was to salvage a place for Marxist thought in the elaborate structure of mathematical knowledge. Presented mainly in the form of popular essays, it was generally disregarded by the representatives of modern systems of logic.

Marx made no reference to dialectical logic, either as a special discipline or as a distinct component of his general theory. Soviet philosophers did not have a generally accepted definition of its domain of inquiry. Some identified it as a "general methodology" or as a "methodology of practical activity." Others called it "a science of the most general laws in the development of science, society and human thought." B. M. Kedrov identified it as "subjective dialectics," as a dialectical method in its application to the processes of cognition. All representatives of dialectical logic sought little help from mathematical or other logics and all refrained from taking issues with the ideas and interests of Soviet experts in these fields. Their attacks were limited to Western philosophies, typified by neopositivism, which dealt with the place of logic in modern scientific and philosophical thought and with the role, of linguistics in logic and mathematics. Even this criticism tended to be fortuitous and overly generalized.

The changing attitudes of Marxist philosophers toward the mathematical and philosophical views of N. N. Luzin provided a graphic example of the fundamental changes in the social status and academic autonomy of the Soviet community of mathematicians. In the mid-1930s, Soviet philosophers and other ideologues interpreted Lebesgue's complimentary remark about Luzin's philosophical acumen with great trepidation and alarm. They read it as clear evidence of Luzin's affiliation with effectivism advanced by the Paris school in set theory—as outright surrender to a philosophy dominated by Machian epistemology. Combining this "evidence" with an assortment of alleged unpatriotic deeds, they subjected Luzin to public vituperation that bordered on hysteria. A long procession of defenders of the Stalinist version of Marxist orthodoxy attacked the recognized leader of the most powerful and productive school of Soviet mathematics.

In the 1980s, a new generation of Soviet philosophers and general commentators on mathematics reexamined Lebesgue's statement in light of the ongoing intellectual fermentation. This time the interpretation was radically different: Lebesgue's flattering comment appeared as authoritative recognition of the broad sweep and intellectual depth of Luzin's involvement in modern mathematical exploration. The philosophers came to recognize what mathematicians knew all along.

In the mid-1930s, Marxist commentators on developments in mathematics were quick to chastise Luzin for his intimate ties with religious philosophers, an oasis of anti-Marxist activity. During the 1980s, the same kind of experts found the written correspondence between Luzin and Pavel Flerovskii, a religious philosopher with a solid background in mathematics, to be a source of insightful views on the methodological aspects of mathematics and a signif cant index of the brilliance of both men. Luzin's published letters to various representatives of Western mathematical thought showed both the unusual depth of his mind and the critical bent of his philosophical outlook. Moreover, new studies produced suff cient evidence to present Luzin as one of the primary architects of constructivism as an offshoot of intuitionism and a favored Soviet orientation in the foundations of mathematics [Burova 1976, 104–105; Panov 1987, 270–276]. At least one study counted him among the leading Soviet mathematicians who contributed to the advancement of the Marxist–Leninist interpretation of the fundamental principles of mathematics [Protopopov 1983, 5].

After the publication of Marx's mathematical papers in 1933, the more ambitious Soviet philosophers asserted that the time had come for fruitful and solid work on a Marxist orientation in the foundations of mathematics. Fifty years later, Iu. K. Protopopov [1983, 3] noted that the country was still without a systematic dialectical-materialistic study of the foundations of mathematical analysis, the f eld dealt with in Marx's papers.

In the Soviet Union, one of the basic functions of philosophy was to serve as a bridge between Marxist ideology and science. In the Stalin era, Marxist philosophers performed their professional duty by advising mathematicians how to analyze and interpret the foundational, epistemological, historical, and sociological aspects of their science. They waged an open war on indiscretions they considered particularly injurious to Marxist ideology and Soviet politics. If this war was not as devastating as in the case of biology, the reasons must be sought in the mitigating effects of the resistance of the community of mathematicians to unwanted external interference with the normal f ow of professional work. This resistance, which also included tactical or nominal concessions to selected pressures, produced an "unexpected" development: set theory, whose underlying philosophy stood at the opposite pole from Marxist thought, became the leading and most successful scientif c engagement of Soviet mathematicians. Cantor, the father of set heory, made his identif cation with idealistic philosophy clear when he asserted that the development of mathematical ideas was an immanent process unrestrained by external interference [Dauben 1979, 132–133]. In other words, mathematics is a product of ideas that do not have and do not need an empirical origin.

Stalinist philosophers were steadfast in their insistence on making mathematics fully subservient to Communist policies and state ideology. E. Kol'man spoke for the leading Soviet philosophers when he warned mathematicians against any effort to restore the traditional attributes of professional autonomy. Post-Stalinist philosophers, in contrast, took it for granted that autonomy was a vital and indispensable attribute of the scientif c community in performing professional duties. However, while the Stalinist regime was unsuccessful in eliminating all the prerogatives of autonomy, the post-Stalinist regime allowed only a limited restoration of autonomy. Post-Stalinist writers referred to the new "autonomy" as "relative independence" [Morozov, Petrov, & Perminov 1977, 77]. This development was strong enough to contribute to a gradual and irretrivable erosion of the Stalinist system of political–ideological control.

A full reversal took place in the interpretation of intuition as a source of scientif c ideas, Under Stalin. intuition was treated as a digression from rational thinking and an inevitable path to mysticism [German 1935, 39]. In the more favorable atmosphere of the thaw it was recognized as a potent "nonrational" source of new ideas in the development of science. According to one writer: "Intuition widens the scope of human thought and supplements discursive cognition, especially in the process of creating turning points in the development of science [Dmitriev 1970, 320].

Other changes also helped to soften blows by the defenders of Marxist orthodoxy. As in the past, mathematics was a vital source of national prestige measured by contributions to science. The work of Soviet mathematicians, particularly of those involved in disciplines aff liated with set theory, attracted the attention of professional circles outside the Soviet Union. Kolmogorov, P. S. Aleksandrov, Luzin, Khinchin, and A. I. Mal'tsev were only a few names on the long list of Soviet scholars who made solid contributions to the substance and the spirit of modern mathamatics. Soviet authorities, aware of mathematics as a national treasure, were careful not to carry their war on ideological and philosophical digressions to an extreme. Another militating factor was the scarcity of defenders of orthodox Marxist positions in mathematics.

Despite the moderating conditions and effects, the Stalinist era was marked by the extensive rights enjoyed by doctrinaires in interpreting and protecting the ideological purity and philosophical unity of mathematics, and in exposing nonconforming mathematicians to slanderous attacks. The doctrinaires had the upper hand in appraising the social usefulness and cultural value of individual branches of mathematics.

In the post-Stalin era, the interests of the philosophers of mathematics, as interpreters of Marxist canons, underwent far-reaching changes. Philosophers now addressed themselves not to mathematicians but to the general public. They concentrated on the contributions of mathematics to a modern world outlook and paid special attention to foundational questions, the general role of mathematics in the rapid progress of modern science, and the socio-cultural underpinnings of mathematical knowledge. In pursuing their new duties, philosophers were granted more latitude in interpreting the deeper meanings of mathematical ideas. The emphasis was less on the rigidity of Marxist doctrine than on the general f uidity and unsettledness in current mathematical thought. In the Stalin era, Soviet philosophers both made a futile effort to formulate a Marxist orientation in the foundations of mathematics and totally rejected formalism, logicism, and intuitionism. In the post-Stalin era they readily admitted that the Soviet constructive orientation left unanswered some of the questions related to the foundations of mathematics and that all the leading Western orientations offered valuable and irrefutable—but partial—contributions [Bocharov *et al.* 1979, 107].

Under mounting pressure from the scientif c community, Soviet authorities allowed, in cautious steps, for considerable f exibility in Marxist philosophy that eliminated previously rigid boundaries separating Soviet "materialism" from Western "idealism." This was a time of general Soviet acceptance not only of Einstein's science but also of his epistemological views, as well as Bohr's complementarity principle and Heisenberg's uncertainty principle. This was also a time of favorable Soviet responses to all new developments on the Western mathematical front. Beginning with the 1960s, the government encountered another kind of pressure: the growing dissident movement among scientists—particularly in the younger generation—in search of structural changes in the Soviet political system that would provide constitutional safeguards for free and independent thought. Undisposed to allow for basic changes in the political system, the government made little effort—before the onrushing stream of *perestroika*—to refrain from keeping a vigil eye on the political behavior of scientists and continued to apply various techniques to penalize individuals violating the rigid code of loyalty to state ideology.

In general, however, the post-Stalinist thaw created more favorable conditions for both scientif c work and philosophical engagement. In the 1980s, it was openly admitted that dialectical materialism alone cannot meet all the philosophical needs of mathematics and of science in general. Under Stalin, Engels's *Dialectics of Nature* and *Ani-Dühring* provided unchallengeable guides for a general orientation in mathematics; in 1991, the journal *Priroda* (*Nature*) published a Russian translation of an article by Jean van Heijenroot who assembled enough arguments to conclude that Engels knew little about contemporary mathematics, that his "materialism" was barely more than "crude empiricism," and that his dialectics was a "degenerate and oversimplif ed" notion of Hegel's dialectics [van Kheinrot 1991, 105].

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